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The Momentum Four-Vector in Brans–Dicke Wormholes

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In this work, in order to compute energy and momentum distributions (due to matter plus fields including gravitation) associated with the Brans–Dicke wormhole solutions we consider Møller's energy-momentum complexes both in general relativity and the teleparallel gravity, and the Einstein energy-momentum formulation in general relativity. We find exactly the same energy and momentum in three of the formulations. The results obtained in teleparallel gravity is also independent of the teleparallel dimensionless coupling parameter, which means that it is valid not only in the teleparallel equivalent of general relativity, but also in any teleparallel model. Furthermore, our results also sustains (a) the importance of the energy-momentum definitions in the evaluation of the energy distribution of a given spacetime and (b) the viewpoint of Lessner that the Møller energy-momentum complex is a powerful concept of energy and momentum. (c) The results calculated supports the hypothesis by Cooperstock that the energy is confined to the region of non-vanishing energy-momentum tensor of matter and all non-gravitational fields.

KEY WORDS: energy; Brans Dicke wormhole; teleparallel gravity.

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1. INTRODUCTION

The first locally conserved energy-momentum formulation was constructed by Einstein (1915). Consequently, several energy-momentum prescriptions have been proposed (Bergmann and Thomson, 1953; Landau and Lifshitz, 2002; Møller, 1958, 1961a,b; Papapetrou, 1948; Qadir and Sharif, 1952; Tolman, 1934; Weinberg, 1972). Except for the Møller definition these formulations only give meaningful results if the calculations are performed in *Cartesian* coordinates. Møller proposed a new expression for energy-momentum complex which could

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be utilized to any coordinate system. Next, Lessner (1996) argued that the Møller prescription is a powerful concept for energy-momentum in general relativity. This approach was abandoned for a long time due to severe criticism for a number of reasons (Bergqvist, 1972a,b; Chandrasekhar and Ferrari, 1991; Chen and Nester, 1999). Virbhadra and collaborators revived the interest in this approach (Aguirregabiria *et al.*, 1996; Rosen and Virbhadra, 1993, 1995; Virbhadra, 1990a,b,c; 1991, 1992, 1995a,b, 1999; Virbhadra and Parikh, 1993, 1994a,b) and since then numerous works on evaluating the energy and momentum distributions of several gravitational backgrounds have been completed (Grammenos and Radinschi, gr-qc/0602105; Radinschi, 1999, 2000a,b, 2001, 2002, 2004; Vagenas, 2003a,b, 2004, 2005; gr-qc/0602107; Xulu, 1998, 2000a,b,c, 2003; Yang and Radinschi, 2003). Later attempts to deal with this problematic issue were made by proposers of quasi-local approach. The determination as well as the computation of the quasi-local energy and quasi-local angular momentum of a $(2 + 1)$-dimensional gravitational background were first presented by Brown *et al.* (1994). A large number of attempts since then have been performed to give new definitions of quasi-local energy in Einstein's theory of general relativity (Brown *et al.*, 1997; Hayward, 1994; Hawking and Horowitz, 1996; Liu and Yau, 2003; Yau, 2001). Furthermore, according to the Cooperstock hypothesis (Cooperstock, 1992, 1993, 1997, 2000), the energy is confined to the region of non-vanishing energy-momentum tensor of matter and all non-gravitational fields.

Recently, the problem of energy-momentum localization has also been considered in teleparallel gravity (Nashed, 2002; Vargas, 2004). Møller showed that a tetrad description of a gravitational field equation allows a more satisfactory treatment of the energy-momentum complex than does general relativity. Therefore, we have also applied the super-potential method by Mikhail *et al.* (1993) to calculate the energy of the central gravitating body. In (Nashed, 2002; Vargas, 2004); Vargas, using the definitions of Einstein and Landau–Lifshitz in teleparallel gravity, found that the total energy is zero in Friedmann–Robertson–Walker space-times. There are also several papers on the energy-momentum problem in teleparallel gravity. The authors obtained the same energy-momentum for different formulations in teleparallel gravity (Aydogdu, 2006a,b; Aydogdu *et al.*, 2005; Aydogdu and Saltı, 2006; Havare *et al.*, 2006; Saltı, 2005a,b,c, 2006; Saltı and Aydogdu, 2006; Saltı and Havare, 2005). Considerable efforts have also been performed in constructing super-energy tensors (Senovilla, 2000). Motivated by the works of Bel (1958, 1960, 1962) and independently of Robinson (Robinson, 1997), many investigations have been carried out in this field (Bonilla and Senovilla, 1997; Garecki, 2001; Lazkog *et al.*, 2003).

The paper is organized as follows. In the next section, we introduce the Brans–Dicke wormhole solutions. In Section 3, we calculate energy-momentum in general relativity using Møller and Einstein's energy-momentum prescriptions. Next, in Section 4, we find energy-momentum in Møller's tetrad theory of gravity.

Finally, Section 5 is devoted to conclusions. *Notations and conventions*: $c = G = 1$, metric signature $(-, +, +, +)$, Greek indices run from 0 to 3 and, Latin ones from 1 to 3. Throughout this paper, Latin indices $(i, j, \ldots)$ number the vectors, and Greek indices $(\mu, \nu, \ldots)$ represent the vector components.

2. THE BRANS–DICKE WORMHOLE SOLUTIONS

There is a revival of interest in the Brans–Dicke theory due principally to the following reasons: The theory occurs naturally in the low energy limit of the effective string theory in four dimensions or the Kaluza–Klein theory. It is found to be consistent not only with the weak field solar system tests but also with the recent cosmological observations. Moreover, the theory accommodates Mach's principle (It is known that Einstein's theory of general relativity cannot accommodate Mach's principle satisfactorily) (Nandi and Zhang, 2006).

A less well known yet an important area where the Brans–Dicke theory has found immense applications is the field of wormhole physics, a field recently re-activated by the seminal work of Morris and colleagues (Morris and Thorne, 1988; Morris *et al.*, 1988). Wormholes are topological handles that connect two distant regions of space. These objects are invoked in the investigations of problems ranging local to cosmological scales, not to mention the possibility of using these objects as a means of interstellar travel. Wormholes require for their construction what is called "exotic matter"—matter that violate some or all of the known energy conditions, the weakest being the averaged null energy condition. Such matters are known to arise in quantum effects. However, the strongest theoretical justification for the existence of exotic matter comes from the notion of dark energy or phantom energy that are necessary to explain the present acceleration of the universe (Nandi and Zhang, 2004, 2006; Nandi *et al.*, 1998, 2004).

The string action, keeping only linear terms in the string tension α and in the curvature R, takes the following form in the matter free region ($S_{\mathrm{Matter}} = 0$):

$$S_{\mathrm{String}} = \frac{1}{\alpha} \int d^4x \sqrt{-g} e^{-2\Phi} \left[R + 4 g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi \right] \tag{1}$$

where $g^{\mu,\nu}$ is the string metric and Φ is the dilaton field. Note that the zero values of other matter fields do not impose any additional constraints either on the metric or on the dilaton (Kar, 1999). Under the substitution $e^{-2\Phi} = \psi$, the above action reduces to the Brans–Dicke action

$$S_{\mathrm{Brans\text{–}Dicke}} = \int d^4x \sqrt{-g} \left[\psi R + \frac{1}{\psi} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi \right] \tag{2}$$

in which the Brans–Dicke coupling parameter $\omega = -1$. This particular value is actually model independent and it actually arises due to the target space duality. It should be noted that the Brans–Dicke action has a conformal invariance characterized by a constant gauge parameter ζ (Cho, 1992). Arbitrary values of can actually lead to a shift from the value $\omega = -1$, but fix this ambiguity by choosing $\zeta = 0$.

Under a further substitution

$$\tilde{g}_{\mu\nu} = \psi g_{\mu\nu} \tag{3}$$

$$d\varphi = \sqrt{\frac{2\omega + 3}{2\alpha'}} \frac{d\psi}{\psi}, \quad \alpha' \neq 0, \quad \omega \neq \frac{3}{2} \tag{4}$$

in which we have introduced, on purpose, a constant parameter $\alpha' = 0$ that can have any sign. Then the Brans–Dicke action transforms into the form of the Einstein minimally coupled scalar field theory (EMCSFT) action (Nandi and Zhang, 2006)

$$S_{\text{EMCSFT}} = \int d^4x \sqrt{-g} \left[\psi R + \frac{1}{\psi} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi \right] \tag{5}$$

The EMCSFT equations are given by

$$R_{\mu\nu} = -\alpha' \partial_\mu \varphi \partial \nu \varphi \tag{6}$$

$$\varphi^{\mu}_{;\mu} = 0 \tag{7}$$

Let us now consider the class I EMCSFT solution due to Buchdahl (1959)

$$\begin{aligned} ds^2 = &- \left(1 - \frac{m}{2r}\right)^{2\beta} \left(1 + \frac{m}{2r}\right)^{-2\beta} dt^2 \\ &+ \left(1 - \frac{m}{2r}\right)^{2(1-\beta)} \left(1 + \frac{m}{2r}\right)^{2(1+\beta)} [dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \end{aligned} \tag{8}$$

and we also have

$$\varphi(r) = \sqrt{\frac{2(\beta^2 - 1)}{\alpha'}} \ln \left[\left(1 - \frac{m}{2r}\right) \left(1 + \frac{m}{2r}\right)^{-1} \right] \tag{9}$$

where m and β are two arbitrary constants. The metric can be expanded to give

$$\begin{aligned} ds^2 = &- \left(1 - \frac{2M}{r} + \frac{2M^2}{r^2} + O\left(\frac{1}{r^3}\right)\right) dt^2 \\ &+ \left(1 - \frac{2M}{r} + O\left(\frac{1}{r^3}\right)\right) [dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \end{aligned} \tag{10}$$

from which one can read of the Keplerian mass $M = m\beta$. The metric has a naked singularity at $r = \frac{m}{2}$. For $\beta = 1$, it reduces to the Schwarzschild solution in isotropic coordinates. For $\alpha' = 1$ and $\beta > 1$, it represents a traversable wormhole. It is symmetric under inversion of the radial coordinate $r \to r^{-1}$ and we have two asymptotically flat regions (at $r = 0$ and $r = \infty$) connected by the throat occurring at $r_0^+ = \frac{m}{2}[\beta + \sqrt{\beta^2 - 1}]$. Thus real throat is guaranteed by $\beta^2 > 1$. For the choice $\alpha' = 1$, the quantity $\sqrt{2(\beta^2 - 1)}$ is real such that there is a real scalar charge σ from the line-element (8) given by

$$\varphi = \frac{\sigma}{r} = -\frac{2m}{r}\sqrt{\frac{\beta^2 - 1}{2}} \tag{11}$$

But in this case, we have violated almost all energy conditions in importing *by hand a* negative sign before the kinetic term in equations (6) and (7). Alternatively, we could have chosen $\alpha' = -1$ in equation (8), giving an imaginary charge σ. In both cases, however, we end up with the same equation $R_{\mu\nu} = -\partial_\mu\varphi\partial_\nu\varphi$. There is absolutely no problem in accommodating an imaginary scalar charge in the wormhole configuration (Nandi and Zhang, 2004, 2006; Nandi *et al.*, 1998, 2004; Armendáriz-Pícon, 2002).

3. FOUR-MOMENTUM IN GENERAL RELATIVITY

The aim of this part of the study is to evaluate energy distribution associated with the black holes given above. The Møller and Einstein energy-momentum complexes will be considered.

In general relativity, Møller's energy-momentum complex is given by (Møller, 1958, 1961a,b)

$$\Omega_\mu^\nu = \frac{1}{8\pi}\frac{\partial \chi_\mu^{\nu\alpha}}{\partial x^\alpha} \tag{12}$$

satisfying the local conservation laws:

$$\frac{\partial \Omega_\mu^\nu}{\partial x^\nu} = 0 \tag{13}$$

where the antisymmetric super-potential χ_μ^ν is

$$\chi_\mu^{\nu\alpha} = \sqrt{-g}\left\{\frac{\partial g_{\mu\beta}}{\partial x^\gamma} - \frac{\partial g_{\mu\gamma}}{\partial x^\beta}\right\} g^{\nu\gamma} g^{\alpha\beta}. \tag{14}$$

The locally conserved energy-momentum complex Ω_μ^ν contains contributions from the matter, non-gravitational and gravitational fields. Ω_0^0 is the energy density and Ω_a^0 are the momentum density components. The momentum four-vector definition of Møller is given by

$$\int\int\int \Omega_\mu^0 dxdydz. \tag{15}$$

Using Gauss's theorem, this definition transforms into

$$P_\mu = \frac{1}{8\pi} \int\int \chi_\mu^{0a} \mu_a d/S \tag{16}$$

where μ_a (where $a = 1, 2, 3$) is the outward unit normal vector over the infinitesimal surface element dS. P_i give momentum components P_1, P_2, P_3 and P_0 gives the energy.

For the space-time of Brans–Dicke wormhole, $g_{\mu\nu}$, is defined by

$$g_{\mu\nu} = -\left(1 - \frac{2M}{r} + \frac{2M^2}{r^2} + O\left(\frac{1}{r^3}\right)\right)\delta_\mu^0\delta_\nu^0 + \left(1 - \frac{2M}{r} + O\left(\frac{1}{r^3}\right)\right)\delta_\mu^1\delta_\nu^1$$
$$+ \left(1 - \frac{2M}{r} + O\left(\frac{1}{r^3}\right)\right) r^2\delta_\mu^2\delta_\nu^2 + \left(1 - \frac{2M}{r} + O\left(\frac{1}{r^3}\right)\right) r^2 \sin^2\theta\delta_\mu^3\delta_\nu^3, \tag{17}$$

and its inverse $g^{\mu\nu}$ is

$$\begin{aligned} g^{\mu\nu} = &-\left(1 - \frac{2M}{r} + \frac{2M^2}{r^2} + O\left(\frac{1}{r^3}\right)\right)^{-1} \delta_0^\mu\delta_0^\nu \\ &+ \left(1 - \frac{2M}{r} + O\left(\frac{1}{r^3}\right)\right)^{-1} \delta_1^\mu\delta_1^\nu \\ &+ \frac{\delta_2^\mu\delta_2^\nu}{2}\left(1 - \frac{2M}{r} + O\left(\frac{1}{r^3}\right)\right)^{-1} \\ &+ \frac{\delta_3^\mu\delta_3^\nu}{r^2\sin^2\theta}\left(1 - \frac{2M}{r} + O\left(\frac{1}{r^3}\right)\right)^{-1}. \end{aligned} \tag{18}$$

The required non-zero component of the super-potential of Møller, for the line-element (10), is

$$\chi_0^{01}(r, \theta) = \frac{2M\sin\theta\left(1 - \frac{4M}{r} + \frac{4M^2}{r^2}\right)}{\sqrt{\frac{1}{r^3}\left(r^3 - 4M^3 - 4Mr^2 + 6M^2r\right)}} \tag{19}$$

while the momentum density distributions take the form

$$\Omega_1^0 = 0, \quad \Omega_2^0 = 0, \quad \Omega_3^0 = 0. \tag{20}$$

Therefore, if we substitute these results into equation (15), we get the total energy that is contained in a *sphere* of radius r

$$E_{\text{MØller}}(r) = \frac{M\left(1 - \frac{4M}{r} + \frac{4M^2}{r^2}\right)}{\sqrt{\frac{1}{r^3}\left(r^3 - 4M^3 - 4Mr^2 + 6M^2r\right)}}$$

$$= \aleph^3 r^{-\frac{1}{2}} - \frac{\aleph}{4} r^{\frac{1}{2}} + \frac{3}{32\aleph} r^{\frac{3}{2}} + O\big(r^{\frac{5}{2}} big) \tag{21}$$

which are also the energy (mass) of the gravitational field that a neutral particle experiences at a finite distance r. Here, we have defined that $\aleph = (-M)^{1/2}$ (remember $M = m\beta$, and m and β are arbitrary constants). Additionally, we can find the momentum components which are given by

$$\vec{P}_{\text{MØller}}(r) = 0. \tag{22}$$

The formulation of Einstein (1915a,b) is defined as

$$\Theta_\mu^\nu = \frac{1}{16\pi} H_{\mu,\alpha}^{\nu\alpha} \tag{23}$$

where

$$H_\mu^{\nu\alpha} = \frac{g_{\mu\beta}}{\sqrt{-g}} [-g(g^{\nu\beta} g^{\alpha\xi} - g^{\alpha\beta} g^{\nu\xi})]_{,\xi} \tag{24}$$

In the equation above, Θ_0^0 is the energy density, Θ_a^0 are the momentum density components, and Θ_0^a are the components of energy-current density. The Einstein energy and momentum density satisfies the local conservation laws

$$\frac{\partial \Theta_\mu^\nu}{\partial x^\nu} = 0 \tag{25}$$

and the energy-momentum components are given by

$$P_\mu = \int\int\int \Theta_\mu^0 dx dy dz. \tag{26}$$

P_μ is called the momentum four-vector, P_a give momentum components P_1, P_2, P_3 and P_0 gives the energy.

In order to use the Einstein energy-momentum complex, we have to transform the line element (10) in quasi-Cartesian coordinates. By using the relations

$$x = r \sin\theta \cos\phi \tag{27}$$

$$y = r \sin\theta \sin\phi, \tag{28}$$

$$z = r \cos\theta, \tag{29}$$

one gets

$$ds^2 = -Adt^2 + D(dx^2 + dy^2 + dz^2) + \frac{B - D}{r^2} (xdx + ydy + zdz)^2 . \tag{30}$$

Where,

$$A = 1 - \frac{2M}{r} + \frac{2M^2}{r^2} + O\left(\frac{1}{r^3}\right), \quad B = D = 1 - \frac{2M}{r} + O\left(\frac{1}{r^3}\right). \tag{31}$$

Using this metric, one gets the following expressions for the energy

$$E_{\text{Einstein}} = M\left(1 - \frac{4M}{r} + \frac{4M^2}{r^2}\right)\left(1 - \frac{4M}{r} + \frac{6M^2}{r^2} - \frac{4M^3}{r^3}\right)^{-\frac{1}{2}}$$
$$= \aleph^3 r^{-\frac{1}{2}} - \frac{\aleph}{4} r^{\frac{1}{2}} + \frac{3}{32\aleph} r^{\frac{3}{2}} + O\left(r^{\frac{5}{2}}\right) \tag{32}$$

and for the momentum components

$$P_1^{(\text{Einstein})} = P_2^{(\text{Einstein})} = P_3^{(\text{Einstein})} = 0. \tag{33}$$

4. FOUR-MOMENTUM IN TELEPARALLEL GRAVITY

In this part of the study, we calculate energy-momentum distribution associated with a given space-time model in Møller's tetrad theory of gravity.

Møller modified general relativity by constructing a new field theory in teleparallel space. The aim of this theory was to overcome the problem of the energy-momentum complex that appears in Riemannian space (Møller, 1961a,b, 1978). The field equations in this new theory were derived from a Lagrangian which is not invariant under local tetrad rotation. Saez (1983) generalized Møller theory into a scalar tetrad theory of gravitation. Meyer (1982) showed that Møller theory is a special case of Poincare gauge theory (Hayashi and Shirafuji, 1980a,b; Hehl *et al.*, 1980).

In a space-time with absolute parallelism the teleparallel vector fields h_i^μ define the non-symmetric connection

$$\Gamma^\alpha_{\mu\beta} = h_i^\alpha h^i_{\mu\beta} \tag{34}$$

where $h^i_{\mu,\beta} = \partial_\beta h^i_\mu$. The curvature tensor which is defined by $\Gamma^\alpha_{\mu\beta}$ is identically vanishing.

Møller constructed a gravitational theory based on this space-time. In this gravitation theory the field variables are the 16 tetrad components h_i^μ from which the metric tensor is defined by

$$g^{\alpha\beta} = h_i^\alpha h_i^\beta \eta^{ij} \tag{35}$$

We assume an imaginary value for the vector h_0^μ in order to have a Lorentz signature. We note that, associated with any tetrad field h_i^μ there is a metric field defined uniquely by Eq. (35), while a given metric $g^{\alpha\beta}$ doesn't determine the tetrad field completely; for any local Lorentz transformation of the tetrads h_i^μ leads to a new set of tetrads which also satisfy Eq. (35). The lagrangian L is an invariant constructed from $\xi_{\alpha\beta\mu}$ and $g^{\alpha\beta}$, where $\xi_{\alpha\beta\mu}$ is the con-torsion tensor given by

$$\xi_{\alpha\beta\mu} = h_{i\alpha} h^i_{\beta;\mu} \tag{36}$$

where the semicolon denotes covariant differentiation with respect to Christoffel symbols

$$\left\{ \begin{array}{c} \alpha \\ \mu\nu \end{array} \right\} = \frac{1}{2} g^{\alpha\beta}(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}) \tag{37}$$

The most general Lagrangian density invariant under the parity operation is given by the following form

$$L = \sqrt{-g}\left(m_1 \Phi^\rho \Phi_\rho + m_2 \xi^{\alpha\beta\mu} + m_3 \xi^{\alpha\beta\mu} \xi_{\alpha\beta\mu}\right) \tag{38}$$

where $g = \det(g_{\alpha\beta})$, and Φ_ρ is the basic vector field defined by

$$\Phi_\mu = \xi^\rho_{\mu\rho} \tag{39}$$

Here, m_1, m_2 and m_3 are constants determined by Møller such that the theory coincides with Einstein's theory of general relativity in the weak fields,

$$m_1 = \frac{-1}{\kappa} \quad m_2 = \frac{\lambda}{\kappa} \quad m_3 = \frac{1}{\kappa}(1 - 2\lambda) \tag{40}$$

where κ is the Einstein constant and λ is a free dimensionless parameter. The same choice of the parameters was also obtained by Hayashi and Nakano (1967).

Møller applied the action principle to the Lagrangian density given by Eq. (38) and obtained the field equation in the form

$$G_{\alpha\beta} + H_{\alpha\beta} = -\kappa T_{\alpha\beta} \tag{41}$$

$$F_{\alpha\beta} = 0 \tag{42}$$

where $G_{\alpha\beta}$ is the Einstein tensor, and defined by

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = G_{\alpha\beta}. \tag{43}$$

Here, $H_{\alpha\beta}$ and $F_{\alpha\beta}$ are given by

$$H_{\alpha\beta} = \lambda\left[\xi_{\mu\nu\alpha}\xi^{\mu\nu}_{\beta} + \xi_{\mu\nu\alpha}\xi^{\mu\nu}_{\beta} + \xi_{\mu\nu\beta}\xi^{\mu\nu}_{\alpha} + g_{\alpha\beta}\left(\xi_{\mu\nu\lambda}\xi^{\lambda\nu\mu} - \frac{1}{2}\xi_{\mu\nu\lambda}\xi^{\mu\nu\lambda}\right)\right] \tag{44}$$

$$F_{\alpha\beta} = \lambda\left[\Phi_{\alpha,\beta} - \Phi_{\beta,\alpha} - \Phi_\rho\left(\xi^\rho_{\alpha\beta} - \xi^\rho_{\beta\alpha}\right) + \xi^\rho_{\alpha\beta;\rho}\right] \tag{45}$$

and they are symmetric and skew symmetric tensors, respectively.

Møller assumed that the energy and momentum tensor of the matter fields is symmetric. In the Hayashi–Nakano theory, however, the energy and momentum tensor of spin-$\frac{1}{2}$ fundamental particles has a non-vanishing anti-symmetric part arising from the effects due to intrinsic spin, and the right-hand side of Eq. (42) doesn't vanish when we take into account the possible effects of the intrinsic spin.

It can be shown (Hayashi and Shirafuji, 1979) that the tensors, $H_{\alpha\beta}$ and $F_{\alpha\beta}$, consist of only those terms which are linear or quadratic in the axial-vector part of the torsion tensor, ζ_ρ, given by

$$\zeta_\rho = \frac{1}{3}\varepsilon_{\rho\mu\lambda}\xi^{\mu\nu\lambda}. \tag{46}$$

Where, $\epsilon_{\rho\mu\nu\lambda}$ is given by following definition

$$\varepsilon_{\rho\mu\nu\lambda} = \sqrt{-g}\delta_{\rho\mu\nu\lambda} \tag{47}$$

and $\delta_{\rho\nu\nu\lambda}$ being completely anti-symmetric and normalized as $\delta_{0123} = -1$. Therefore, both $H_{\alpha\beta}$ and $F_{\alpha\beta}$ vanish if the ζ_ρ is vanishing. In other words, when ζ_ρ is found to vanish from anti-symmetric part of field equations (42) and symmetric part of (41) coincides with the Einstein equation.

The super-potential of Møller's in teleparallel gravity is given by Mikhail *et al.* (1993) as

$$U_\mu^{\nu\beta} = \frac{(-g)^{1/2}}{2\kappa}P_{\chi\rho\sigma}^{\tau\nu\beta}\left[\Phi^\rho g^{\sigma\chi}g_{\mu\tau} - \lambda g_{\tau\mu}\xi^{\chi\rho\sigma} - (1-2\lambda)\,g_{\tau\mu}\xi^{\sigma\rho\chi}\right] \tag{48}$$

with $P_{\zeta\rho\sigma}^{\tau\mu\beta}$ is

$$P_{\chi\rho\sigma}^{\tau\mu\beta} = \delta_\chi^\tau g_{\rho\sigma}^{\nu\beta} + \delta_\rho^\tau g_{\sigma\chi}^{\nu\beta} - \delta_\sigma^\tau g_{\chi\rho}^{\nu\nu} \tag{49}$$

with $g_{\rho\sigma}^{\nu\beta}$ being a tensor defined by

$$g_{\rho\sigma}^{\nu\beta} = \delta_\rho^\nu\delta_\sigma^\beta - \delta_\sigma^\nu\delta_\rho^\beta. \tag{50}$$

The energy-momentum density is defined by Møller (1961a,b, 1978)

$$\Xi_\sigma^\beta = U_{\alpha,\lambda}^{\beta\lambda} \tag{51}$$

where comma denotes ordinary differentiation. The energy E and momentum components P_i are expressed by the volume integral Møller (1961a,b, 1978),

$$E = \lim_{r\to\infty}\int_{r=\text{constant}} \Xi_0^0 dxdydz, \tag{52}$$

$$P_i = \lim_{r\to\infty}\int_{r=\text{constant}} \Xi_i^0 dxdydz, \tag{53}$$

Here, the index of i takes the value from 1 to 3. The angular momentum of a general relativistic system is given by Møller (1961a,b, 1978)

$$J_i = \lim_{r\to\infty}\int_{r=\text{constant}} \left(x_j\,\Xi_k^0 - x_k\,\Xi_j^0\right) dxdydz \tag{54}$$

where i, j and k take cyclic values 1, 2 and 3. We are interested in determining the total energy, and the momentum components.

The general form of the tetrad, h_i^μ, having spherical symmetry was given by Robertson (1932). In the Cartesian form it can be written as

$$h_0^0 = iW, \quad h_a^0 = Zx^a, \quad h_0^\alpha = iHx^\alpha, \quad h_a^\alpha = K\delta_a^\sigma + Sx^a x^\alpha + \varepsilon_{a\alpha\beta} G x^\beta. \tag{55}$$

Where, W, K, Z, H, S, and G are functions of t and $r = \sqrt{x^a x^a}$, and the zeroth vector h_0^μ has the factor $i^2 = -1$ to preserve Lorentz signature, and the tetrad of Minkowski space-time is $h_a^\mu = \mathrm{diag}(i, \delta_a^\alpha)$ where $(a = 1, 2, 3)$. Using the general coordinate transformation

$$h_{a\mu} = \frac{\partial \mathbf{X}^{\nu'}}{\partial \mathbf{X}^\mu} h_{a\nu} \tag{56}$$

where $\{\mathbf{X}^\mu\}$ and $\{\mathbf{X}^{\nu'}\}$ are, respectively, the isotropic and Schwarzschild coordinates (t, r, θ, ϕ). In the spherical, static and isotropic coordinate system $\mathbf{X}^1 = r\ \sin\theta\cos\phi$, $\mathbf{X}^2 = r\sin\theta\sin\phi$, $\mathbf{X}^3 = r\cos\theta$. We obtain the tetrad components of h_a^μ as

$$\begin{pmatrix} \frac{i}{\sqrt{1-\frac{2M}{r}+\frac{2M^2}{r^2}+O\left(\frac{1}{r^3}\right)}} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{1-\frac{2M}{r}+O\left(\frac{1}{r^3}\right)}} s\theta c\phi & \frac{1}{r\sqrt{1-\frac{2M}{r}+O\left(\frac{1}{r^3}\right)}} c\theta c\phi & \frac{s\phi}{r\sqrt{1-\frac{2M}{r}+O\left(\frac{1}{r^3}\right)} s\theta} \\ 0 & \frac{1}{\sqrt{1-\frac{2M}{r}+O\left(\frac{1}{r^3}\right)}} s\theta c\phi & \frac{1}{r\sqrt{1-\frac{2M}{r}+O\left(\frac{1}{r^3}\right)}} c\theta s\phi & \frac{c\phi}{\sqrt{1-\frac{2M}{r}+O\left(\frac{1}{r^3}\right)} s\theta} \\ 0 & \frac{1}{\sqrt{1-\frac{2M}{r}+O\left(\frac{1}{r^3}\right)}} c\theta & \frac{1}{r\sqrt{1-\frac{2M}{r}+O\left(\frac{1}{r^3}\right)}} s\theta & 0 \end{pmatrix} \tag{57}$$

where $i^2 = -1$. Here, we have introduced the following notation: $s\theta = \sin\theta$, $c\theta = \cos\theta$, $s\phi = \sin\phi$ and $c\phi = \cos\phi$. After performing the calculations (TCI Software Research, 2003; Wolfram Research, 2003), the required non-vanishing components of $\xi_{\alpha\beta\mu}$ are found as

$$\xi_{01}^0 = \frac{M(r + 2M)}{r\left(r^2 - 2Mr - 2M^2\right)} \tag{58}$$

$$\xi_{11}^1 = \frac{M}{r^2 - 2Mr} \tag{59}$$

$$\xi_{22}^1 = -\left(\xi_{12}^2\right)^{-1} = \left(\xi_{13}^3\right)^{-1} = \frac{1}{\sin^2\theta}\xi_{33}^1 = -r\sqrt{\frac{r(r-2M)}{r^2 - 2Mr - 2M^2}} \tag{60}$$

$$\xi_{31}^3 = \xi_{21}^2 = \frac{1}{r}\left(1 + \frac{M}{r - 2M}\right) \tag{61}$$

$$\xi_{33}^{2} = -\frac{\sin 2\theta}{2} \tag{62}$$

$$\xi_{23}^{3} = \xi_{32}^{3} = \cot\theta \tag{63}$$

and the non-vanishing components of Φ_μ are

$$\Phi_1 = \frac{M}{r^2 - 2Mr} - \frac{1}{r}\sqrt{\frac{r^2 - 2Mr - 2M^2}{r\,(r - 2M)}}, \qquad \Phi_2 = \cot\theta. \tag{64}$$

Next, we obtain the required Møller's super-potentials of $\sum_{\mu}^{\mu\beta}$ as following

$$\Sigma_{0}^{01} = \frac{2M\sin\theta}{\kappa}\left(1 - \frac{4M}{r} + \frac{4M^2}{r^2}\right)\left(1 - \frac{4M}{r} + \frac{6M^2}{r^2} - \frac{4M^3}{r^3}\right)^{-\frac{1}{2}} \tag{65}$$

while the momentum density distributions take the form

$$\Xi_{1}^{0} = 0, \quad \Xi_{2}^{0} = 0, \quad \Xi_{3}^{0} = 0. \tag{66}$$

Hence, we find the following energy

$$E_{\text{MØller}}^{TP}(r) = \aleph^3 r^{-\frac{1}{2}} - \frac{\aleph}{4} r^{\frac{1}{2}} + \frac{3}{32\aleph} r^{\frac{3}{2}} + O\left(r^{\frac{5}{2}}\right) \tag{67}$$

Here, *TP* means teleparallel gravity, and we have defined again $\aleph = (-M)^{\frac{1}{2}}$. Next, one can easily see that the momentum components are

$$\vec{P}_{\text{MØller}}^{TP}(r) = 0 \tag{68}$$

These results are exactly the same as obtained in the general relativity analog of Møller energy-momentum formulation. It is evident that the teleparallel gravitational results are independent of teleparallel dimensionless coupling parameter λ which means that these results are valid not only in teleparallel equivalent of general relativity but also in any teleparallel model.

5. SUMMARY AND FINAL COMMENTS

The problem of energy-momentum localization has been one of the most interesting and thorny problems which remains unsolved since the advent of Einstein's theory of general relativity and tetrads theory of gravity. Misner *et al.* (1973) argued that the energy is localizable only for spherical systems. Cooperstock and Sarracino (1978) contradicted their viewpoint and argued that if the energy is localizable in spherical systems then it is also localizable for all systems. Bondi (1990) expressed that a non-localizable form of energy is inadmissible in relativity and its location can in principle be found. Cooperstock hypothesized that in a curved space-time energy-momentum is/are confined to the region of non-vanishing energy-momentum tensor $T_{\mu\nu}$ and consequently the gravitational waves are not carriers of energy and/or momentum in vacuum space-times. This hypothesis has neither been proved nor disproved. There are many results that support this hypothesis (Xulu, 2000a,b,c, 2003).

The problem of calculating energy-momentum distribution of the universe has been considered both in Einstein's theory of general relativity and teleparallel theory of gravity. From the advents of these different gravitation theories various methods have been proposed to deduce the conservation laws that characterize the gravitational systems. The first of such attempts was made by Einstein who proposed an expression for the energy-momentum distribution of the gravitational field. There exists an opinion that the energy-momentum definitions are not useful to get finite and meaningful results in a given geometry. Virbhadra and his collaborators re-opened the problem of the energy and momentum by using the energy-momentum complexes. The Einstein energy-momentum complex, used for calculating the energy in general relativistic systems, was followed by many complexes: e.g. Tolman, Papapetrou, Bergmann-Thomson, Møller, Landau-Liftshitz, Weinberg, Qadir-Sharif and the teleparallel gravity analogs of the Einstein, Landau-Lifshitz, Bergmann-Thomson and Møller prescriptions. The energy-momentum complexes give meaningful results when one transforms the line element in quasi-Cartesian coordinates. The energy and momentum complex of Møller gives the possibility to perform the calculations in any coordinate system. To this end Virbhadra and his collaborators have considered many space-time models and have shown that several energy-momentum complexes give the same and acceptable results for a given spacetime. Vargas using the definitions of Einstein and Landau-Lifshitz in teleparallel gravity, found that the total energy is zero in Friedmann–Robertson–Walker spacetimes.

In this study, we have calculated the energy-momentum distributions (due to matter and fields including gravity) of the Brans–Dicke wormhole solutions in general relativity by using Møller and Einstein energy-momentum formulations, and also in Møller's tetrad theory of gravity (the teleparallel geometry). We find the same energy in the three of the techniques. Our results show that the Møller energy-momentum formulation in general relativity and its teleparallel gravitational analog agree with each other. Also we obtained that the momentum components are equal to zero in three of the formulations:

$$E^{TP}_{\text{Møller}} = E_{\text{Møller}}(r) = E_{\text{Einstein}}(r) = \aleph^3 r^{-\frac{1}{2}} + \frac{3}{32\aleph} r^{\frac{3}{2}} + O\left(r^{\frac{5}{2}}\right) \tag{69}$$

$$\vec{P}^{TP}_{\text{Møller}}(r) = \vec{P}_{\text{Møller}}(r) = \vec{P}_{\text{Einstein}}(r) = 0. \tag{70}$$

Next, the teleparallel gravitational results are independent of the teleparallel dimensionless coupling parameter, which means that they are valid in any teleparallel model. Furthermore, this paper sustains (a) the importance of the energy-momentum definitions in the evaluation of the energy distribution of a given space-time, (b) the viewpoint of Lessner that the Møller energy-momentum complex is a powerful concept of energy and momentum, and (c) the hypothesis by Cooperstock that the energy is confined to the region of non-vanishing energy-momentum tensor of matter and all the non-gravitational fields.

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